Ch. 3. Viscosity and Surface tension

A. Viscosity

1. **Introduction:** The flow of liquid can be classified into two types: i) stream line flow and ii) turbulent flow.

Consider a liquid flowing over a plane surface or through a tube. The velocity of the liquid is less than a certain limiting value called as critical velocity; the flow is called stream line flow. In the stream line flow, the velocity of the liquid at a given point is always constant. During the flow of liquid, it can be imagined to be divided into thin layers, such that in each layer velocity of the liquid has same value at all points. This velocity is proportional to distance of the layer from fixed layer i.e., there is regular velocity gradient setup in the liquid. The greater the distance of a layer from the fixed surface, the greater its velocity, with the topmost layer moving with the fastest.

If the velocity of the liquid is greater than the critical velocity, the flow is no longer stream line flow. It loses all order and becomes disorderly. It is called as turbulent flow. In case of turbulent flow, the velocity of the liquid at any point does not remain constant.

A fluid always opposes the motion of any object passing through it. This is due to a property called viscosity possessed by the fluid. Viscosity is also called fluid friction. Just as the force of friction opposes relative motion between two solids in contact, there exists a force which opposes relative motion between two adjacent layers of fluid. This force is called viscous force. The property possessed by a liquid by virtue of which it opposes relative motion between its different layers is known as **viscosity**.

2. ENERGY OF LIQUID IN MOTION

During the flow a liquid has inertia, it possesses kinetic energy when in motion. It also exerts pressure on the walls of the vessel containing liquid, has pressure energy and may also have potential energy due to its position.

There are three types of energy possessed by a liquid in flow such as i) kinetic energy, ii) potential energy and iii) pressure energy.

i) **Kinetic energy**: We know that, the kinetic energy of a mass *m* of a liquid, flowing with a velocity *v* is given by,

$$kinetic\ energy = \frac{1}{2}mv^2$$

We know that, *density = mass / volume*

For unit volume of the liquid, $m = \rho$ then

The kinetic energy per unit volume of the liquid= $\frac{1}{2}\rho v^2$

For unit mass of the liquid, $m = 1$, therefore

Kinetic energy per unit mass of the liquid =$\frac{1}{2}v^2$

ii) **Potential energy:** Due to position of liquid it has potential energy.

we know that, the potential energy of a liquid of mass *m* at a height *h* above the earth surface is = *mgh*.

If we consider unit volume of the liquid, mass = density i.e., $m = \rho$

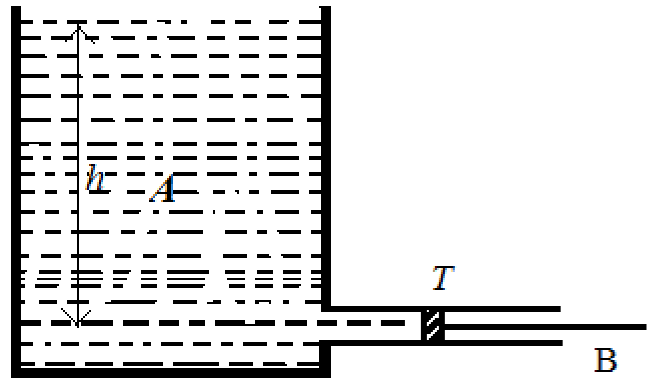
Therefore, potential energy per unit volume of the liquid = ρgh

For unit mass of liquid, m = 1

Potential energy per unit mass of the liquid = gh

iii) Pressure energy:

Consider a tank A containing a liquid of density 'ρ' provided with a narrow side tube T of cross-sectional are 'a' properly fitted with piston B that can be smoothly moved in and out.



Let P be hydrostatic pressure due to liquid at the level of the axis of side tube.

$\therefore$ force on piston =F = $P.\ a$

If more liquid is to be introduced into the tank, this much force has to be applied to the piston in moving inwards.

Let the piston be moving slowly inwards through a distance x due to force, then work has to be done.

The amount of work is to be performed = $F.x = Pax$

This work is required to make the liquid move against pressure P. This becomes energy of mas of the liquid in the tank. It is referred as pressure energy of liquid.

$\therefore$ the pressure energy of liquid = Pax ---------------*(1)*

The volume of liquid = $a.x$

$\therefore$ the mass of liquid forced into tank = $ax\rho$ ---------------*(2)*

The pressure energy per unit mass of liquid = $\frac{Pax}{ax\rho} = \frac{P}{\rho}$

If we consider unit volume of liquid then,

The pressure energy per unit mass of liquid = $\frac{Pax}{ax} = P$

3. Bernoulli's Theorem:

It states that the total energy of a small amounts of liquid flowing from one point to another without any friction remains constant throughout the displacement.

i.e. total energy = constant

K. E. + P. E. + Pressure energy = constant.

$$\frac{1}{2}V^2 + hg + {}^{P}\!/_{\rho} = constant$$

This is known as Bernoulli's theorem.

Proof:

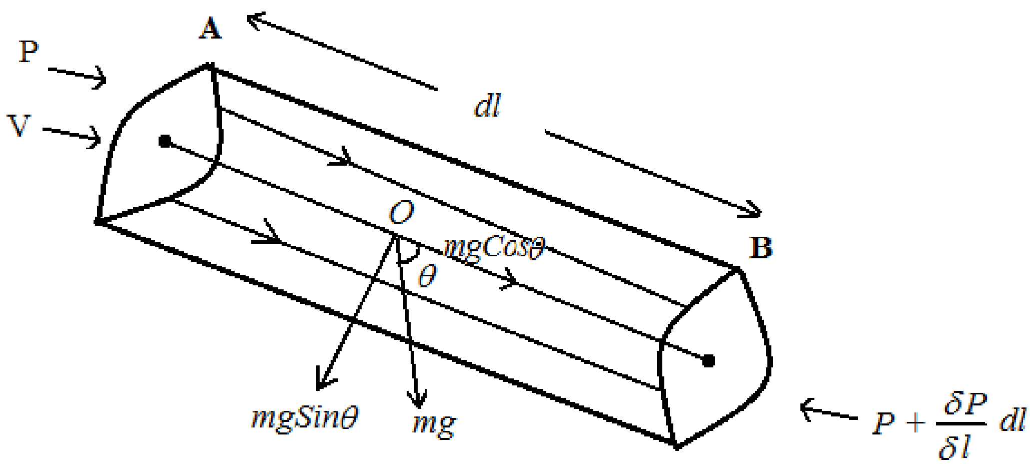


Fig. 1 Folw of non-viscous fluid.

Consider an infinettesimal portio AB of a tube of length dl as shown in Fig.1.

Let *da* be area of cross-section .

Let ρ be density of fluid.

The mass of portion AB , $m = volume \times density = da\ dl\ \rho$

$\therefore$Its weight, $m\ g = da\ dl\ \rho\ g$

Let weight *mg* acts vertically downward at its centre of gravity O making an angle θ with the direction of flow.

The weight *mg* has two component;

i) $mgCos\theta$ acts in a direction of flow.
ii) $mgSin\theta$ acting perpendicular to wall of tube.

Let P be pressure on face A of the tube that on face B will be (P+$\frac{\delta P}{\delta l}dl$) in direction.

$\therefore$ Force acting on face A in forward direction is = *P.da*

and force acting on face B in backward direction is =(P+$\frac{\delta P}{\delta l}dl$)$da$

Net force acting on mass m of fluid is,

$F = Pda - \left(P + \frac{\delta P}{\delta l}dl\right)da + mgCos\theta$

$$= Pda - Pda - \frac{\delta P}{\delta l}dl\,da + mgCos\theta$$

$$= -\frac{\delta P}{\delta l}dl\,da + mgCos\theta$$

$$F = -\frac{\delta P}{\delta l}dl\,da + da\,dl\,\rho\,gCos\theta$$

Let $Cos\theta = -\frac{\delta h}{\delta l}$ where h is vertical height of tube AB from plane.

$$F = -\frac{\delta P}{\delta l}dl\,da - da\,dl\,\rho\,g\frac{\delta h}{\delta l}$$

---------------------- *(1)*

Let *V* is velocity of liquid at face A and *dV/dt* is acceleration.

$\therefore$ force acting on mass *m* of fluid is,

$$force = mass\ \times acceleration = da\,dl\,\rho \times \frac{dV}{dt}$$

---------------------------- *(2)*

Consider velocity is function of both distance and time.

$$dV = \frac{\delta V}{\delta l}.dl + \frac{\delta V}{\delta t}dt$$

$$\frac{dV}{dt} = \frac{\delta V}{\delta l}.\frac{dl}{dt} + \frac{\delta V}{\delta t} = \frac{\delta V}{\delta l}.V + \frac{\delta V}{\delta t}$$

Substituting this value in eq^n (2),

$$F = da\ dl\ \rho \times \left[\frac{\delta V}{\delta l}.V + \frac{\delta V}{\delta t}\right]$$

------------------------ *(3)*

Equating eq^n (1) and eq^n (3)

$$da\ dl\ \rho \times \left[\frac{\delta V}{\delta l}.V + \frac{\delta V}{\delta t}\right] = -\frac{\delta P}{\delta l}dl\ da - da\ dl\ \rho\ g\frac{\delta h}{\delta l}$$

Dividing both side by *da.dl,*

$$\rho \times \left[\frac{\delta V}{\delta l}.V + \frac{\delta V}{\delta t}\right] = -\frac{\delta P}{\delta l} - \rho g\frac{\delta h}{\delta l}$$

--------------------*(4)*

This equation is applicable to both steady and unsteady fluid flow and is referred as Euler's equation.

For steady flow $\frac{\delta V}{\delta t} = 0$

$$\rho V\frac{dV}{dl} = -\frac{dP}{dl} - \rho g\frac{dh}{dl}$$

$$\rho V dV = -dP - \rho g dh$$

$$V dV = -\frac{dP}{\rho} - g dh$$

$$V dV + \frac{dP}{\rho} + g dh = 0$$

Taking integration on both sides

$$\int VdV + \int \frac{dP}{\rho} + \int gdh = 0$$

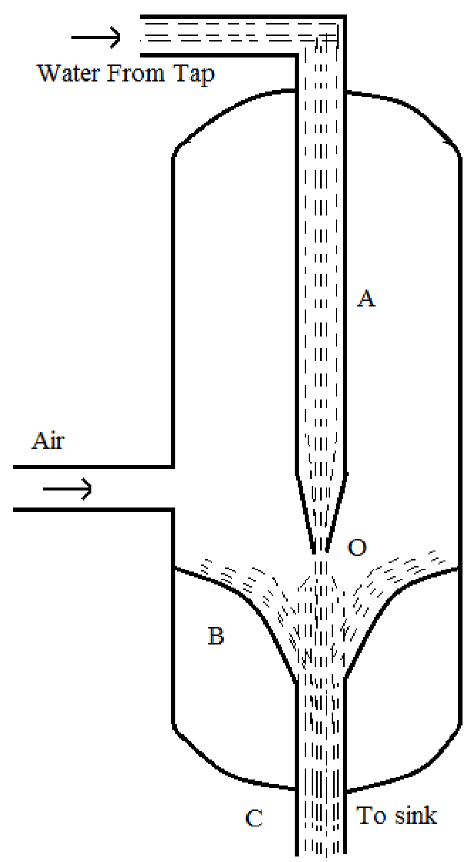
$$\frac{V^2}{2} + \frac{P}{\rho} + gh = Constant$$

$$K.\,E + P.\,E. + Pressure\ energy = constant$$

This is proof of Bernoulli's theorem.

4. Practical Applications of Bernoulli's Theorem:

i) Filter Pump:



It is based on the Principle of Bernoulli's theorem and is used to reduce the pressure in a vessel. A stream of water from a tap flowing through a tube A, issues out in the form of a jet from its narrow orifice O. It will result in a great rise in its velocity and proportionate fall in its pressure. The pressure is reduced to a value below that of atmosphere.

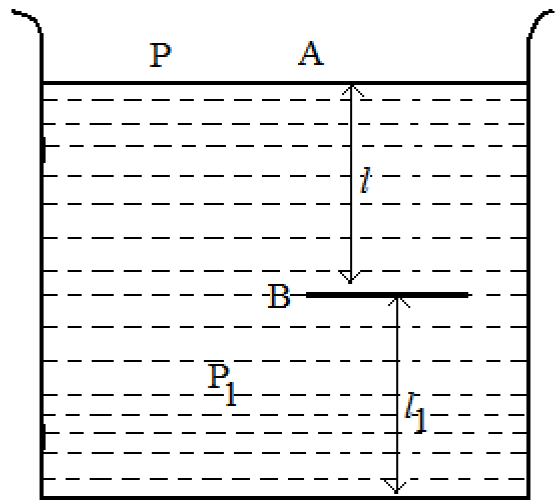
The air from the vessel connected through a side tube B to this region of reduced pressure then rushes into it and is carried away by the stream of water as it flows down through C.

In this way, the pressure in the vessel is ultimately reduced to just a little above the vapour pressure of water, in a comparetively very short time.

If the inlet water tube be twisted, instead of straight one, the exhaustion proceeds more rapidly due to rotating water-jet in the tube breaking up more readily and mixing up easily with the incoming air from the vessel.

ii) **Law of Hydrostatic Pressure:**

It is the pressure exerted by liquid at equilibrium at a given point due to the force of gravity. It is called the hydrostatic pressure.



Hydrostatic pressure increases in proportion to depth measured from the surface increases. As increasing weight of liquid exerting downward force from above.

Consider the liquid having density ρ inserted in the tank. Let A be a point on the surface of liquid at a distance $(l + l_1)$ from ground surface. Let P be atmospheric pressure at point A. Let B be a point at a distance l_1 from ground surface. Let P_1 be pressure at point B.

By using Bernoulli's theorem,

$$\frac{V^2}{2} + \frac{P}{\rho} + gh = Constant$$

At point A and B total energy is remains constant.

$$\frac{V_A^2}{2} + \frac{P_A}{\rho} + gh_A = \frac{V_B^2}{2} + \frac{P_B}{\rho} + gh_B = Constant$$

Since liquid is in equilibrium $V_A = V_B = 0$

At point A, $P_A = P$ be atmospheric pressure and $h_A = l+l_1$

At point B, $P_B = P_1$ and $h_B = l_1$

$$\frac{P}{\rho} + g(l + l_1) = \frac{P_1}{\rho} + gl_1$$

$$\frac{P}{\rho} + gl + gl_1 = \frac{P_1}{\rho} + gl_1$$

$$\frac{P}{\rho} + gl = \frac{P_1}{\rho}$$

$$P_1 = P + \rho g l$$

This shows that the pressure inside the liquid increases with the depth of liquid increases.

5. Newton's law of Viscosity:

It states that viscous force acting tangentially on any liquid layer is directly proportional to its surface area *A* and velocity gradient *dV/dx*.

$$\therefore\ F \propto A.\frac{dV}{dx}$$

$$\therefore F = -\eta A\frac{dV}{dx}$$

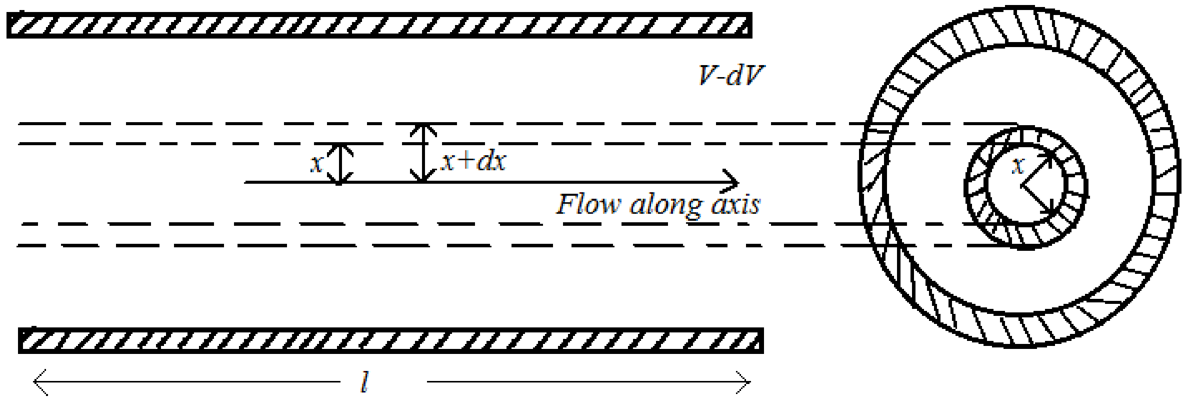
where –Ve sign indicate that direction of force is opposite to velocity gradient. η is constant known as coefficient of viscosity.

If $A = 1$ sq.cm and $dV/dx = 1$ then $F = \eta$.

Thus, the coefficient of viscosity of a liquid may be defined as the tangential force required per unit area to maintain unit velocity gradient.

CGS unit of coefficient of viscosity is Poise or dyne-sec/cm^2.

6. Poiseuille's equation for flow of liquid through capillary tube:



Consider stream line flow of liquid through a capillary tube of radius *r* placed horizontal. When a liquid flow through the tube in stream line nature, it forms coaxial cylindrical layers through the cylindrical tube, the layer at axis of tube moving with fastest velocity and velocity of successive layers going on decreasing as layers approach the inner surface of tube. The layer in contact with the inner surface of tube is practically stationary with velocity zero. Also, there is constant velocity gradient between the layers.

Consider layers at distance x and $x+dx$ from axis of tube as shown in Fig. Since velocity gradient between these layers is dV/dx.

The retarding force on layer at distance x due to layer x+dx is, $F = -\eta A \frac{dV}{dx}$ -----(1)

Where η is coefficient of viscosity, A is cross-sectional area of tube, l is length of tube.

As tube is cylindrical with radius x and length l

Area of tube = $2\pi x l$

From eq^n (1), $F = -\eta . 2\pi x l \frac{dV}{dx}$ ---------------------- (2)

This retarding force is overcome by accelerating force of pressure P applied between two ends of tube.

The force on layer *x* is,

F = P × Circular cross-sectional area of radius x

= P.πx^2 ------------------------------ (3)

Equating eq^n (2) and eq^n(3)

$$-\eta . 2\pi x l \frac{dV}{dx} = P\pi x^2$$

$$\frac{dV}{dx} = -\frac{Px}{2\eta l}$$

$$dV = -\frac{Px}{2\eta l} dx$$

Integrating on both sides,

$$\int dV = -\frac{P}{2\eta l} \int x \, dx$$

$$V = -\frac{P}{2\eta l} . \frac{x^2}{2} + k$$

$$V = -\frac{Px^2}{4\eta l} + k - - - - - - - (4)$$

Where *k* is constant of integration.

Now $V=0$ when $x = r$, because the layers in contact with sides of tubes are stationary.

$$0 = -\frac{Pr^2}{4\eta l} + k$$

$$k = \frac{Pr^2}{4\eta l}$$

Putting value of k in eq^n (4),

$$V = -\frac{Px^2}{4\eta l} + \frac{Pr^2}{4\eta l} = \frac{P}{4\eta l}(r^2 - x^2) \qquad \text{---------------} (5)$$

This gives velocity of layer at distance *x* from the axis of tube.

Imagine another co-axial cylindrical shell of the liquid of radius (*x*+*dx*).

The volume of liquid flowing per second through cross-sectional area is,

$$dv = area \times velocity = 2\pi x dx.V$$

Substituting value of *V* from eq^n (5),

$$dv = 2\pi x dx.\frac{P}{4\eta l}(r^2 - x^2) = \frac{P\pi}{2\eta l}(xr^2 - x^3)dx - - - - - - (6)$$

Total volume of liquid coming out through tube can be obtained, by integrating eq^n (6) between the limit 0 to r,

$$\int dv = \frac{P\pi}{2\eta l}\int_0^r (r^2 x - x^3)dx$$

$$v = \frac{P\pi}{2\eta l}\left[r^2.\frac{x^2}{2} - \frac{x^4}{4}\right]_0^r$$

$$= \frac{P\pi}{2\eta l}\left[\frac{r^4}{2} - \frac{r^4}{4}\right] = \frac{P\pi}{2\eta l}.\frac{r^4}{4}$$

$$v = \frac{P\pi r^4}{8\eta l} - - - - - - - - (7)$$

Eq^n (7) is called as Poiseuille's equation for rate of flow of liquid through narrow tube.

Coefficient of viscosity is

$$\eta = \frac{P\pi r^4}{8vl}$$

B. Surface Tension

1. Introduction: A liquid at rest shows very interesting property called as surface tension. We have seen that water spider walks on the surface steady of water, a needle floats on the steady surface of water, rain drops and soap bubble always take spherical shape etc. All these phenomena arise due to surface tension. Surface tension is one of the most important properties of liquid.

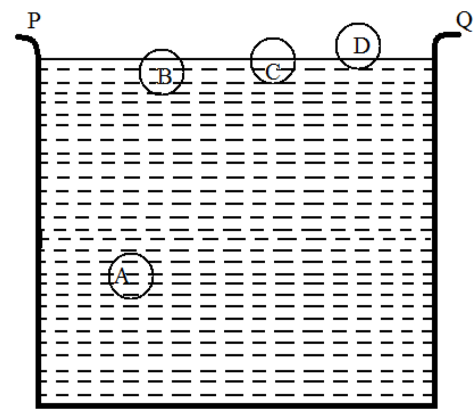
Definition: Surface tension is defined as the force per unit length of a line drawn in the liquid surface acting perpendicular to it at every point and tending to pull the surface apart along the line.

Let F be force acting on imaginary line of length *l*, then surface tension is

$$T = \frac{F}{l}$$

Its dimension is [$M^1L^0T^{-2}$]. SI unit is N/m and CGS unit is dyne/cm.

2. Explanation of Surface tension:



Consider four molecules A, B, C and D of a liquid, with their spheres of influence drawn around them as shown in figure. Sphere A being well inside the liquid; B near the free surface of the liquid; C just on the free surface of the liquid and D above the free surface.

Since the sphere of influence of molecule A, lies wholly inside the liquid, it is attracted equally in all directions by other molecules lying within its sphere of influence, so that there is no resultant cohesive force on it.

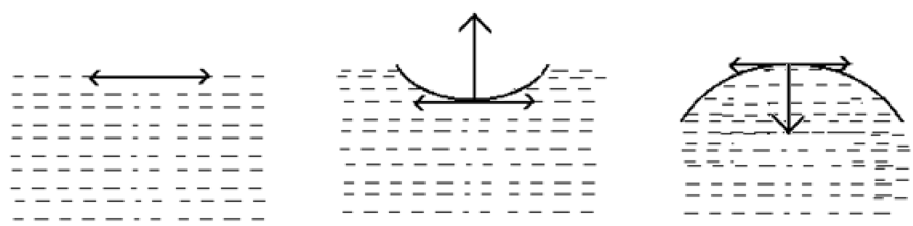
The sphere of influence of molecule B, lies partly outside the liquid and so that the upper half of the sphere contains fewer molecules attracting it upwards, than the lower half, attracting it downwards and the resultant downward force acting on B.

The sphere of influence of molecule C lies on the surface of the liquid, so that full one half of its sphere of influence lies above the surface of the liquid, contain few air molecules whereas there are liquid molecules in its entire lower half and the resultant downward force in this case is maximum. This downward force exerted per unit area of a liquid surface is called internal cohesion pressure.

In the case of molecule D, which is passed out of the liquid surface, only a part of the sphere of influence lies inside the liquid, so that downward force decreases. When the sphere of influences passes entirely outside the liquid surface, there is no downward force on the molecule at all. This shows that all over the surface of liquid there is a downward pull due to attraction between the molecules.

Thus, all the molecules in the surface film are acted upon by an unbalanced net cohesive force directed into the liquid. Thus, molecules in the surface film are pulled inside the liquid. This minimizes the total number of molecules in the surface film. As a result, surface film remains under tension. The surface film of liquid behaves like a stretched elastic membrane. This tension is known as surface tension. The force due to surface tension acts tangentially to the free surface of liquid.

3. Pressure difference across a liquid Surface:



(i) (ii) (iii)

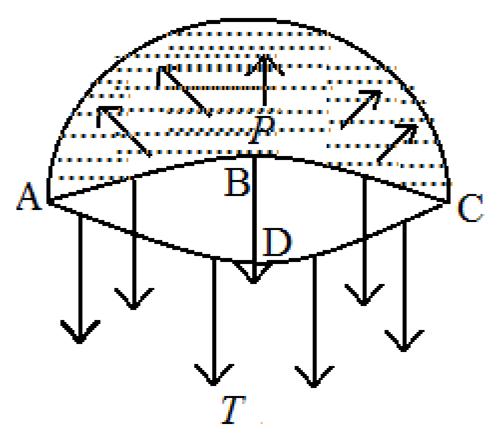
i) Suppose the free surface of a liquid is plane as shown in Fig. (i). Then, the resultant force due to surface tension on a molecule on its surface is zero and cohesion pressure is negligible.

ii) If the free surface of the liquid be concave as shown in Fig. (ii). Then, the resultant force due to surface tension on a molecule would be upwards and cohesion pressure is decreased.

iii) If the liquid surface be convex as shown in Fig. (iii). Then, the resultant force due to surface tension on a molecule will be directed downwards so that cohesion pressure is increased.

4. Excess pressure inside a liquid drop:

Consider a liquid drop of radius *r*. Due to surface tension, molecules lying on the surface of liquid drop will experience a resultant force inwards, perpendicular to the surface. The pressure inside the drop must be greater than the pressure outside it. The excess pressure inside the drop provides a



force outward, perpendicular to the surface to counter balance the resultant force due to surface tension.

Let P_O be pressure outside the liquid drop and P_i be pressure inside the drop.

The excess pressure inside the drop = $(P_i\text{-}P_o)$

Let T be surface tension of the liquid

Consider equilibrium of one half of the drop i.e., upper half or hemisphere, the upward thrust on the plane face ABCD due to excess pressure $(P_i\text{-}P_o)$ is given by

$= (P_i - P_o) \times Area\ of\ circular\ face = (P_i - P_o) \times \pi r^2$ ---------------*(1)*

The force due to surface tension acting downwards on it and round its edge, is given by, = $T\times$ *Circumference of circular face* $=T\times 2\pi r$ ---------------------- *(2)*

Since the hemisphere is in equilibrium, the two forces must be equal. Hence,

$$(P_i - P_o) \times \pi r^2 = T \times 2\pi r$$

$$(P_i - P_o) = \frac{T \times 2\pi r}{\pi r^2}$$

$$(P_i - P_o) = \frac{2T}{r} \text{ ----------------------- (3)}$$

Eq^n(3) gives the expression of excess pressure inside the liquid drop.

5. **Excess pressure inside a soap bubble**:

Consider a soap bubble of spherical shape having radius *r* and surface tension *T*. There are two surfaces of soap bubble. Due to surface tension the molecules on the surface film experience a net force in inward direction normal to the surface. Therefore, there is more pressure inside than outside it.

Let P_O be pressure outside the liquid drop and P_i be pressure inside the drop.

The excess pressure inside the drop = $(P_i\text{-}P_o)$

The excess pressure $(P_i\text{-}P_o)$ is given by

$= (P_i - P_o) \times Area\ of\ circular\ face = (P_i - P_o) \times \pi r^2$ ---------------*(1)*

The force due to surface tension acting downwards on it and round its edge, is given by, = *T*× *Circumference of circular face* =*T*×2π*r*

Since soap bubble has two surfaces, therefore force due to surface tension is, =2× *T*×2π*r* = 4 π*rT*-------------------- (2)

For equilibrium, the two forces must be equal. Hence,

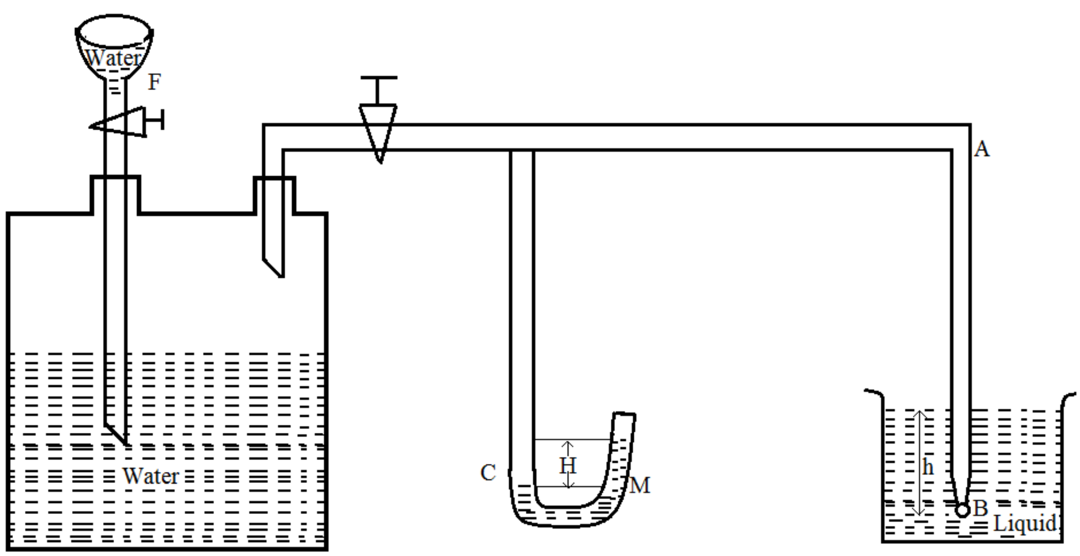
$$(P_i - P_o) \times \pi r^2 = 4\pi r T$$

$$(P_i - P_o) = \frac{4\pi r T}{\pi r^2}$$

$$(P_i - P_o) = \frac{4T}{r}$$

This gives the expression of excess pressure inside the soap bubble.

6. Determination of surface tension by Jaeger's method:



The experimental set up of Jaeger's method to determine surface tension as shown in figure. It consists of a long thin glass tube 'AB', with its lower portion ending in a fine jet and with its tip cut quite smooth and square so as to be perpendicular to its axis. This dips in the experimental liquid. It is then connected to a manometer ‘M’ and Woulff’s bottle, fitted with a dropping funnel ‘F’ containing water. The liquid used in manometer M is Xylol (a liquid of hydrocarbon) in preference of water because of its lower density, so that the difference of level in the two limbs may be large.

Due to capillary action liquid rises up into the tube AB, the shapes of its meniscus being hemispherical. Some air is forced into the tube by dropping water into Woulff’s bottle, which displaces its own volume of air from it. The liquid column in AB slowly moves down until it reaches B, when a bubble is formed there. The radius of curvature of bubble gradually decreases with increasing pressure in it, until it reaches the minimum value and bubble acquires a more or less hemispherical shape with radius r equal to that of aperture at B. The pressure inside being the maximum as indicated by difference of levels (H) in the two limbs of manometer.

The bubble becomes unstable for any further growth of it tends to increase its radius which results in a corresponding decrease in the pressure inside it due to surface tension. Thus, destroying the equilibrium between its internal and constant external pressure, it gets detached from the tube and whole process starts all over again.

Now, clearly just before the bubble breaks away from B, the pressure inside it is equal to pressure at that ‘C’ and is equal to $P+H\rho g$

Where P is atmospheric pressure and $(H\rho g)$ is pressure due to liquid column H in the manometer. ρ be its density.

When bubble just breaks away from B, the pressure on it is equal to that at the level of B in beaker i.e. equal to $P+hdg$

Where h is depth of tip B from surface of liquid in the beaker and d its density.

Therefore, the excess pressure inside the bubble

$$= (P + H\rho g) - (P + hdg)$$

$$= g(H\rho - hd)$$

But the excess pressure inside the bubble,

$$= \frac{2T}{r}$$

$$\therefore \frac{2T}{r} = g(H\rho - hd)$$

$$T = \frac{rg(H\rho - hd)}{2}$$

By using this formula surface tension of liquid is determined.

Numerical:

1. Calculate the rate of flow of water through a horizontal capillary tube of bore radius 0.5mm and length 50cm, when a pressure difference equivalent to height of 50 cm of water column is maintained between ends of tube. If viscosity of water is 10^{-3} Ns/m^2.

Solution: Given

$r = 0.5\text{mm} = 0.5\times10^{-3}$ m

$l = 50$ cm $= 50\times10^{-2}$ m

$h = 50$ cm $= 50\times10^{-2}$ m

$\eta = 10^{-3}$ Ns/m^2

Let $P = hdg = 50\times10^{-2} \times1000 \times9.8 = 490 \times 10 = 4900$ N/m^2

Rate of flow of liquid through a capillary tube is

$$v = \frac{P\pi r^4}{8\eta l}$$

$$= \frac{4900 \times 3.14 \times (5 \times 10^{-4})^4}{8 \times 10^{-3} \times 50 \times 10^{-2}}$$

$$= \frac{15386 \times 625 \times 10^{-16}}{400 \times 10^{-5}}$$

$$= \frac{1.5386 \times 6.25 \times 10^{-10}}{4 \times 10^{-3}}$$

$$= 2.404 \times 10^{-7} m^3/s$$

2. Calculate the rate of flow of water through a horizontal capillary tube of bore radius 0.03cm and length 0.7m. The pressure difference is equivalent to height of 10^2cm of water column is maintained between ends of tube. If viscosity of liquid is 10^{-3} Ns/m^2.

Solution: Given

$r = 0.03\text{cm} = 3\times10^{-4}$ m

$l = 0.7$ m

$h = 10^2$ cm = 1m

$\eta = 10^{-3}$ Ns/m^2

Let P = hdg = 1 ×1000 ×9.8 = 9800 N/m^2

Rate of flow of liquid through a capillary tube is

$$v = \frac{P\pi r^4}{8\eta l}$$

$$= \frac{9800 \times 3.14 \times (3 \times 10^{-4})^4}{8 \times 10^{-3} \times 0.7}$$

$$= \frac{9.8 \times 3.14 \times 10^3 \times 81 \times 10^{-16}}{5.6 \times 10^{-3}}$$

$$= \frac{2492.532}{5.6} \times 10^{-10} = 445.1 \times 10^{-10}$$

$$v = 4.451 \times 10^{-8}\ m^3/s$$

3. Water is escaping from a container by way of a capillary tube, 10cm long and 0.4mm diameter, at a distance of 50cm below the free surface of water in 000container. If the rate of flow of water through a capillary tube is 3.082×10^{-8} m^3/s. Calculate coefficient of viscosity of a liquid.

Solution: Given

$2r = 0.4mm = 4\times10^{-4}$ m

$r = 2\times10^{-4}$ m

$l = 10cm = 0.1m$

$h = 50$ cm $= 0.5m$

$v = 3.082\times10^{-8}\ m^3/s$

$\eta = ?$

Let $P = hdg = 0.5\times1000\times9.8 = 4900\ N/m^2$

Rate of flow of liquid through a capillary tube is

$$v = \frac{P\pi r^4}{8\eta l}$$

Coefficient of viscosity of liquid is,

$$\eta = \frac{P\pi r^4}{8vl}$$

$$= \frac{4900 \times 3.14 \times (2 \times 10^{-4})^4}{8 \times 3.082 \times 10^{-8} \times 0.1}$$

$$= \frac{15386 \times 16 \times 10^{-16}}{24.656 \times 10^{-9}}$$

$$= \frac{2.4618 \times 10^{5} \times 10^{-16}}{24.656 \times 10^{-9}}$$

$= 0.0984 \times 10^{-2} = 9.84 \times 10^{-4}$ Ns/m^2

4. If surface tension of liquid is 6.2×10^{-2}N/m and radius of liquid drop is 10^{-4} m find excess pressure in spherical drop.

   Solution: Given

   $T = 6.2\times10^{-2}$N/m

   $r = 10^{-4}$ m

   We know that, excess pressure in spherical drop is given by

$$P = \frac{2T}{r}$$

$$= \frac{2 \times 6.2 \times 10^{-2}}{10^{-4}}$$

$$= 12.4 \times 10^{2} = 1.24 \times 10^{3} N/m^{2}$$

5. A small hollow sphere which has a small hole in it is immersed in water to a depth of 40 cm before any water penetrates into it. If the surface tension of water is 73 dynes/cm, find the radius of the hole. (g = 980 cm/s^2).

Solution: Given

h= 40cm

T = 73 dynes/cm

g = 980 cm/s^2

r = ?

Let P = hdg = 40×1×980 = 39200 dyne/cm^2

We know that, excess pressure is given by,

$$P = \frac{2T}{r}$$

$$r = \frac{2T}{P} = \frac{2 \times 73}{39200} = \frac{146}{39200} = 0.00372\ cm$$

6. Calculate the surface tension of soap solution, if the pressure inside soap bubble 110 dyne/cm^2 of radius 1cm.

   Solution: Given

   P = 110 dyne/cm^2

   r = 1 cm

   T = ?

   Excess pressure inside soap bubble is,

$$P = \frac{4T}{r}$$

$$T = \frac{Pr}{4} = \frac{110 \times 1}{4} = 27.5\ dyne/cm$$

Multiple Choice Question:

1. In stream line flow of fluids, the velocity of liquid is--------

   a. Remains same throughout motion

   b. Continuously changes with motion

   c. Both a and b   d. none of these

2. In turbulent flow, velocity of liquid is ------ at every point.

   a. same   **b. different**   c. both a and b   d. none of these

3. The property of a liquid by virtue of which it opposes relative motion between its different layers is known as --------

a. Surface tension b. Elasticity

c. Viscosity d. Coefficient of viscosity

4. Which one of the following is not a unit of viscosity?

a) Pa-s b)N-s/m^2 c) Poise **d) Stokes**

5. When liquid flows through the tube then it possesses the energy,

a. Kinetic energy b. Potential energy

c. pressure energy **d. All of these**

6. Kinetic energy per unit volume of liquid is,

a. $\frac{1}{2}\rho v^2$ b. $\frac{1}{2} V^2$ c. ρgh d. $\frac{1}{2} mV^2$

7. Pressure energy per unit mass of liquid is --------

a. P/m *b. m/P* ***c. P/ρ*** *d. P*

8. According to Bernoulli's theorem, the total energy of a small amount of liquid flowing from one point to another without any friction is ------

a. Different at different point **b. remains constant**

c. Increases d. decreases

9. Which of the following is Bernoulli's equation?

a. $hg + \frac{P}{\rho} = Constant$ b. $hg - \frac{P}{\rho} = Constant$

c. $\boldsymbol{hg + \frac{P}{\rho} + \frac{1}{2}v^2 = Constant}$ d. $hg + \frac{P}{\rho} = \frac{1}{2}v^2$

10. Filter pump is used to generate-----

a. Electricity **b. Vacuum** c. Force d. Velocity

11. Rain drops are spherical in shape because of

(a) Surface tension (b) Capillary

(c) Downward motion (d) Acceleration due to gravity

12. Out of the following, which one is not an example of capillary action?

(a) Ploughing of the field

(b) Absorption of ink in a blotting paper

(c) Floating of wood on the surface of water

(d) Rise of oil in the wick of a lamp

13. The surface of water in contact with glass wall is

(a) Plane (b) concave

(c) convex (d) Both 'b' and 'c'

14. A soap bubble of d mm diameter is observed inside a bucket of water. If the pressure inside the bubble is $0.075 \times 10^4 N/m^2$, what will be the value of d? (Take surface tension as 0.075 N/m)

a) 0.4 **b) 0.8** c) 1.6 d) 4

15. A liquid jet of 5 cm diameter has a pressure difference of N/m^2. (Take surface tension as 0.075 N/m)

a) 12 b) 6 c) 3 **d) 1.5**

16. The unit of surface tension in the CGS system is ____________

a) N/m b) Kg/cm **c) dynes/cm** d) dynes/m

17. What is the relationship between excess pressure and the radius of the bubble?

a. are equal in value **b. inversely proportional to each other**

c. directly proportional to each other d. no relationship

18. The total energy of the liquid, if the potential energy is 4.9×10^{-5} J, pressure energy is 1×10^{-5} J and Kinetic energy is 0.016×10^{-5} J ------

a. 4.916×10^{-5} J b. 5.916×10^{-15} J **c. 5.916×10^{-5} J** d. 5.916 J

19. Poiseuille's equation for rate of flow of liquid through a capillary tube of radius *r* is -------------

a. $\boldsymbol{v = \frac{P\pi r^4}{8\eta l}}$ b. $v = \frac{P\pi r^2}{8\eta l}$

c. $v = \frac{P\pi r^3}{2\eta l}$ d. $v = \frac{P\pi r^5}{2\eta l}$

20. What is the property of a liquid due to which its free surface tries to have minimum surface area?

a. Viscosity b. Buoyancy **c. Surface Tension** d. Rigidity

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